AIN SHAMS UNIVERSITY
FACULY OF ENGINEERING

Fluid Mechanics

CEI 121

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Lecturer (17)

C4-1 "The impulse momentum principle"

After completing this chapter, you should be able to

- Identify the various kinds of forces and moments acting on a control volume.
- Use control volume analysis to determine the forces associated with fluid flow.
- Use control volume analysis to determine the moments caused by fluid flow.

- **Whenever the magnitude or direction of the velocity of a body is changed, a force is required to accomplish the change.**
- **Newton's second law of motion is often used to express this concept in mathematical form; the most common form is**

$$F = ma$$

- **Force equals mass times acceleration. Acceleration is the time rate of change of velocity.**

- **In fluid flow problems, a continuous flow is caused to undergo the acceleration, and a different form of Newton's equation is desirable.**
- **Because acceleration is the time rate of change of velocity can be written as**

$$F = ma = m\frac{\Delta v}{\Delta t}$$

- **The term *m/Δt* can be interpreted as the mass flow rate, that is, the amount of mass flowing in a given amount of time.**

$$F = m\,a = m\,\frac{\Delta U}{\Delta t}$$

$$F = m\,a = m\,\frac{\Delta U}{\Delta t} = \rho\,V\,\frac{\Delta U}{\Delta t}$$

$$F = m\,a = m\,\frac{\Delta U}{\Delta t} = \rho\,V\,\frac{\Delta U}{\Delta t} = \rho\,\frac{V}{\Delta t}\,\Delta U = \rho\,Q\,\Delta U$$

$$\sum F = \rho\,Q\,(U_2 - U_1)$$

The last equation should be applied on each direction individually as follows:

Apply in X direction

$$\sum F_x = \rho\, Q\, (U_{x-out} - U_{x-in})$$

Apply in Y direction

$$\sum F_y = \rho\, Q\, (U_{y-out} - U_{y-in})$$

Apply in Z direction

$$\sum F_z = \rho\, Q\, (U_{z-out} - U_{z-in})$$

Impulse

A force applied over a period of time is called an *IMPULSE*.

$$\boxed{I = F \cdot \Delta t}$$

$$I = F\,\Delta t = m\,a\,\Delta t$$

$$I = F\,\Delta t = m\,a\,\Delta t = m\,\frac{\Delta U}{\cancel{\Delta t}}\,\cancel{\Delta t}$$

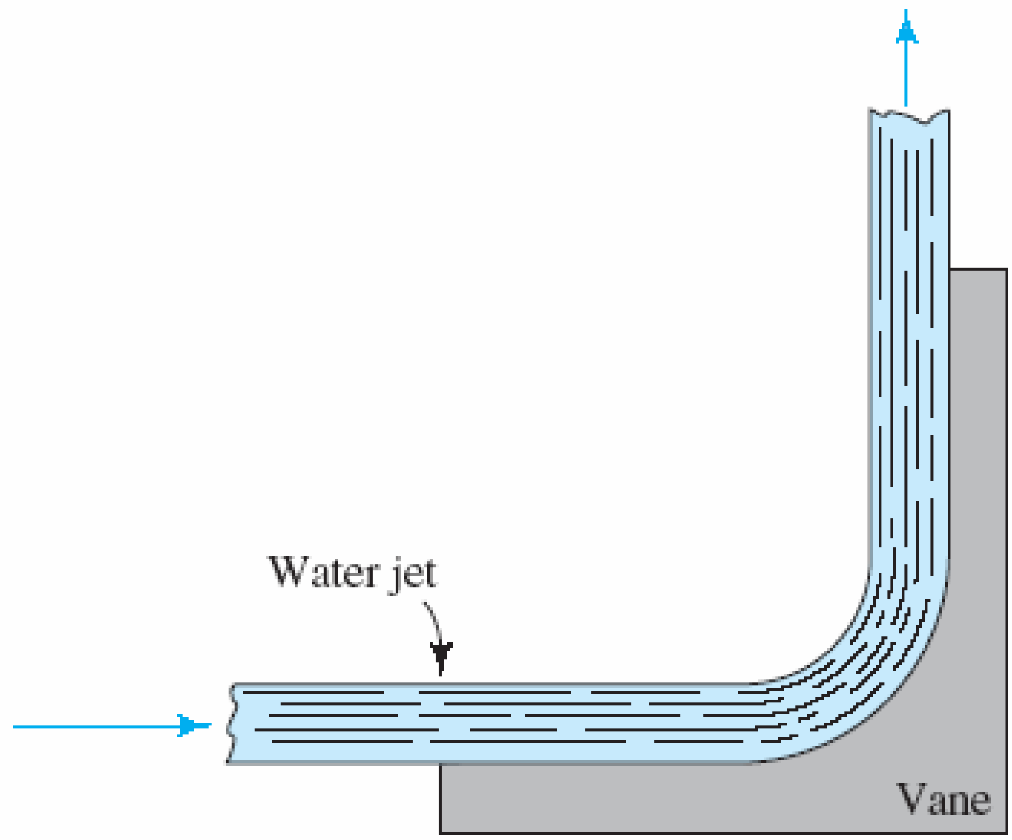
$$I = F\,\Delta t = m\,\Delta U$$

$$I = F\,dt = m\,dU$$

$$I = F\,\Delta t = m\,(U_{out} - U_{in})$$

Example I

A 25-mm-diameter jet of water having a velocity of 6 m/s is deflected 90° by a curved vane, as shown in the below Figure. The jet flows freely in the atmosphere in a horizontal plane. Calculate the *x* and *y* forces exerted on the water by the vane.



Apply the equation in X direction

$$\sum F_x = \rho\, Q\, (U_{x-out} - U_{x-in})$$

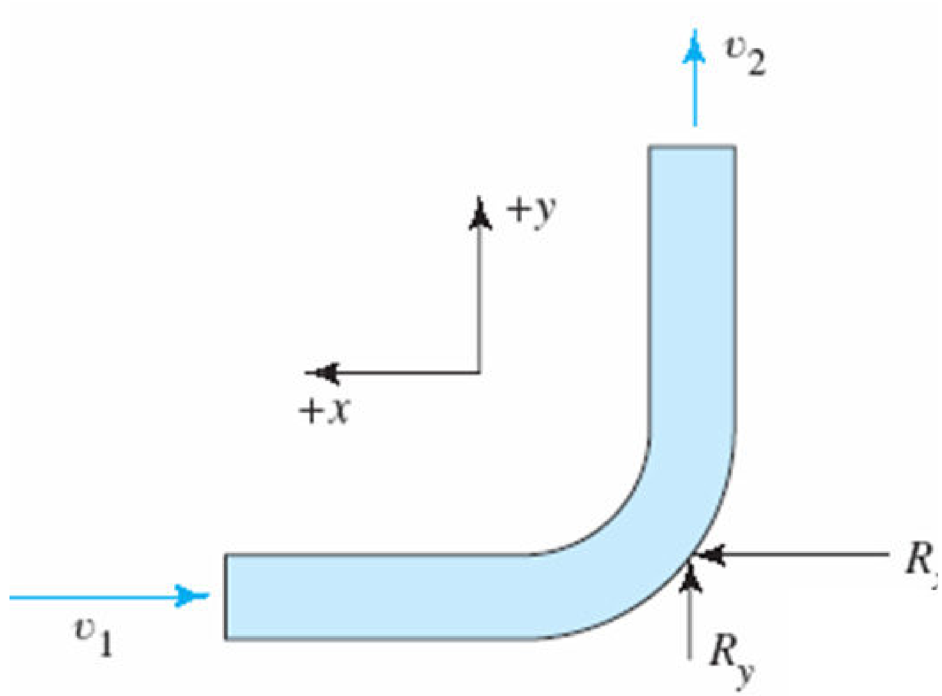
$$\sum F_x = R_x$$

$$\rho = 1000\ kg/m^3$$

$$Q = 0.0005 * 6 = 0.003 m^3/s$$

$$U_{x-in} = -6 m/sec$$

$$U_{x-out} = 0\ m/sec$$



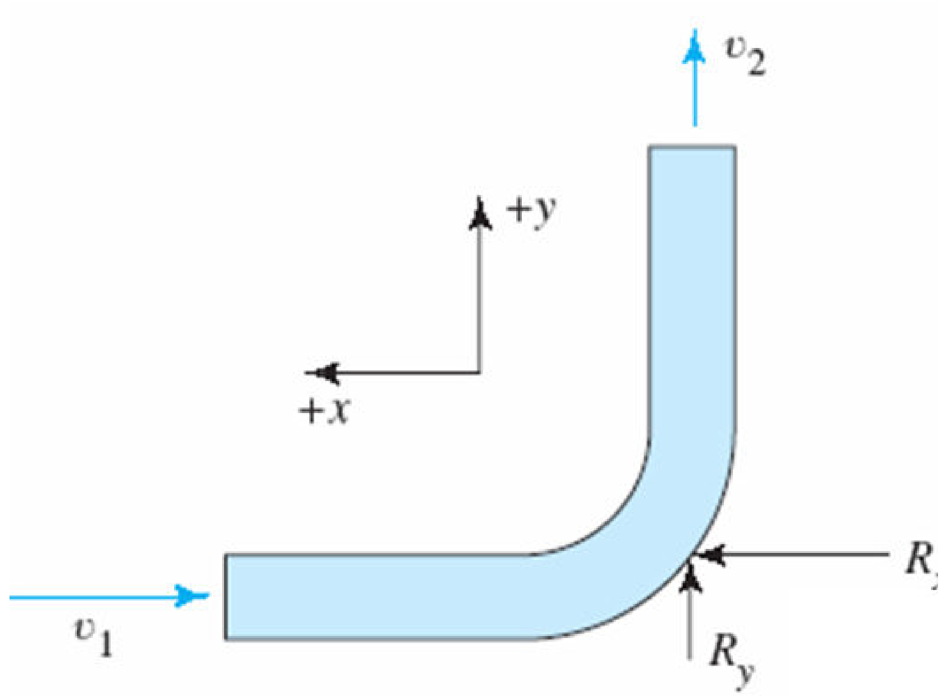
$$\sum F_x = R_x = 1000 * 0.003 *[\,0 - (-6)]$$

$$R_x = 18\ N$$

Apply the equation in Y direction

$$\sum F_y = \rho\, Q\, (U_{y-out} - U_{y-in})$$

$$\sum F_y = R_y$$



$$\rho = 1000\ kg/m^3$$

$$Q = 0.0005 * 6 = 0.003 m^3/s$$

$$U_{y-in} = 0\ m/sec$$

$$U_{y-out} = 6\ m/sec$$

$$\sum F_y = R_y = 1000 * 0.003 * [6 - 0]$$

$$R_y = 18\ N$$

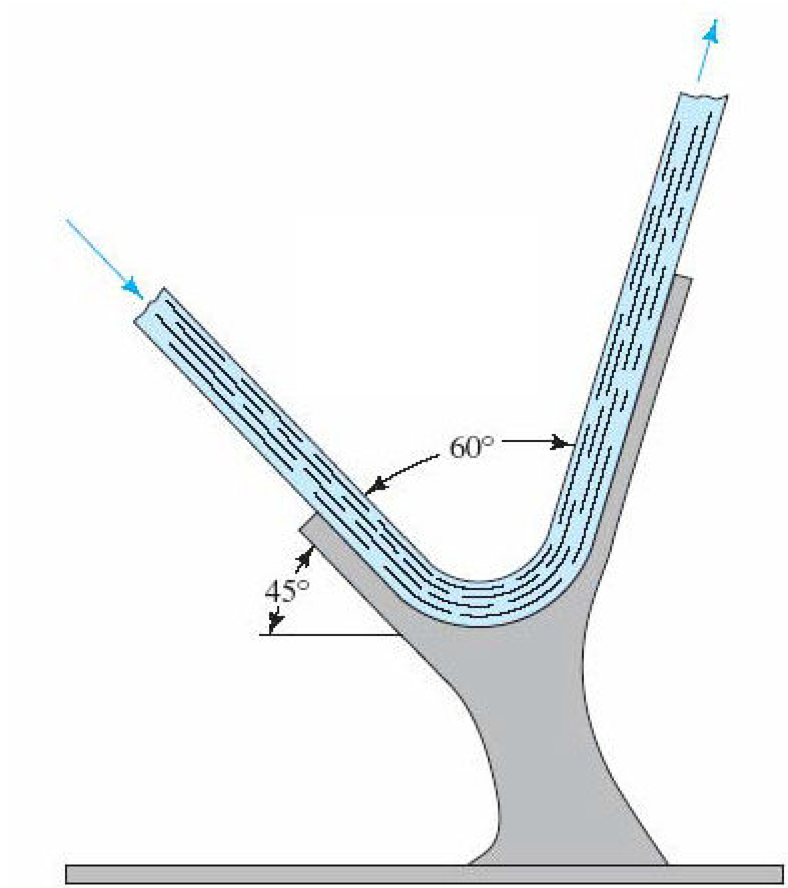
Example II

In a decorative fountain, 0.05 m3/s of water having a velocity of 8 m/s is being deflected by the angled chute shown in the below Figure.

It is required to:

- **Determine the reactions on the chute in the *x* and *y* directions shown.**
- **calculate the total resultant force and the direction in which it acts.**

Neglect elevation changes and the friction of the chute.



Solution

Neglecting the friction of the chute
$V_1 = V_2 = 8 m/s$

Apply the equation in X direction

$$\sum F_x = \rho\, Q\,(V_{x-out} - V_{x-in})$$

$$\rho = 1000\ kg/m^3$$

$$Q = 0.05\ m^3/s$$

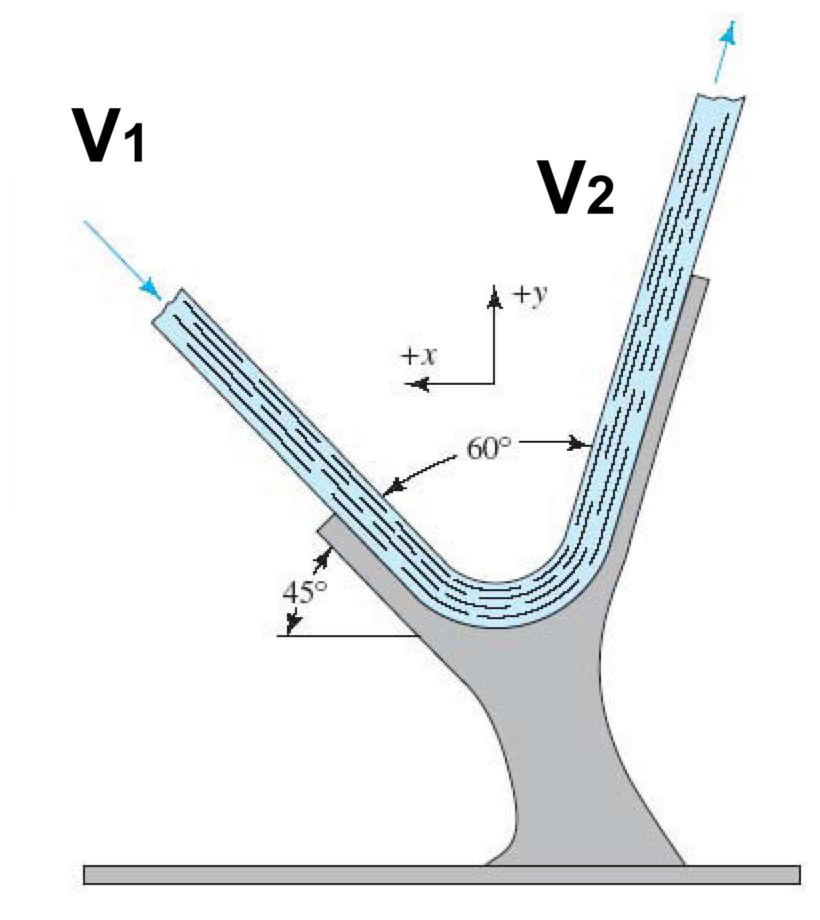
$$U_{x-in} = -8\,Sin\,45 = -5.66\ m/s$$

$$U_{x-out} = -8\,Sin\,15 = -2.07\ m/s$$

$$F_x = 1000 * 0.05(-2.07 + 5.66)$$

$$F_x = 1000 * 0.05(-2.07 + 5.66)$$

$$F_x = 179.5\ N$$



Apply the equation in Y direction

$$\sum F_x = \rho\, Q\, (V_{y-out} - V_{y-in})$$

$\rho = 1000\ kg/m^3$

$Q = 0.05\ m^3/s$

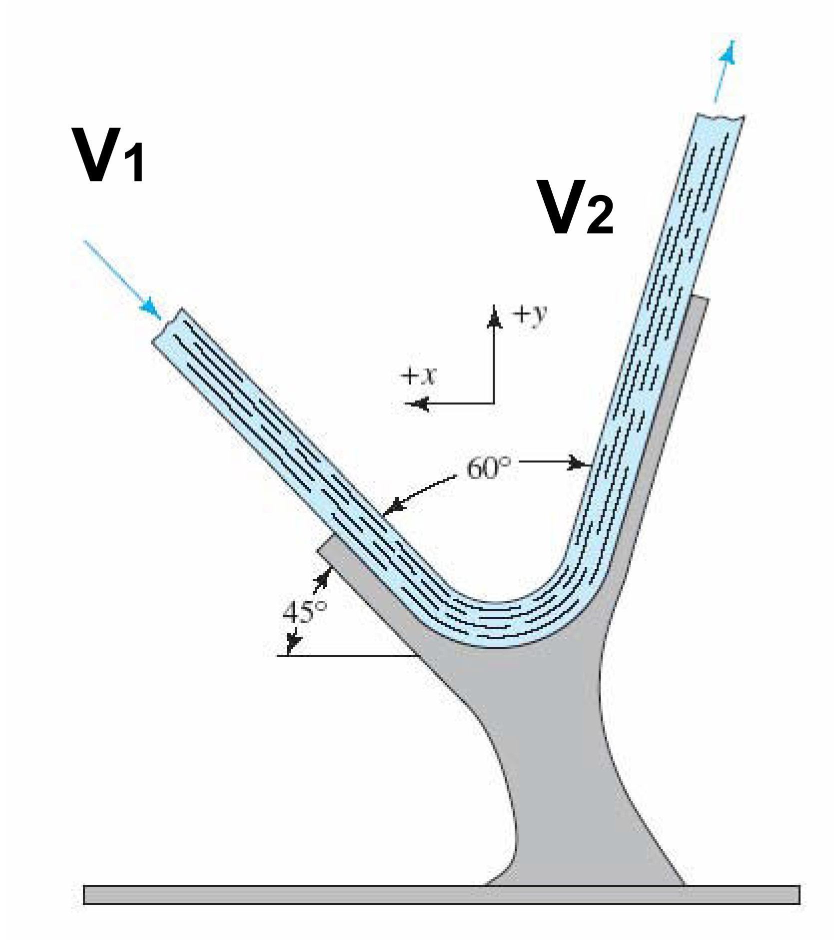
$U_{y-in} = -8\ Cos\ 45 = -5.66$ m/s

$U_{y-out} = 8\ Cos\ 15 = 7.73$ m/s

$F_y = 1000 * 0.05(7.73 + 5.66)$

$F_y = 669.5$ N

The resultant force

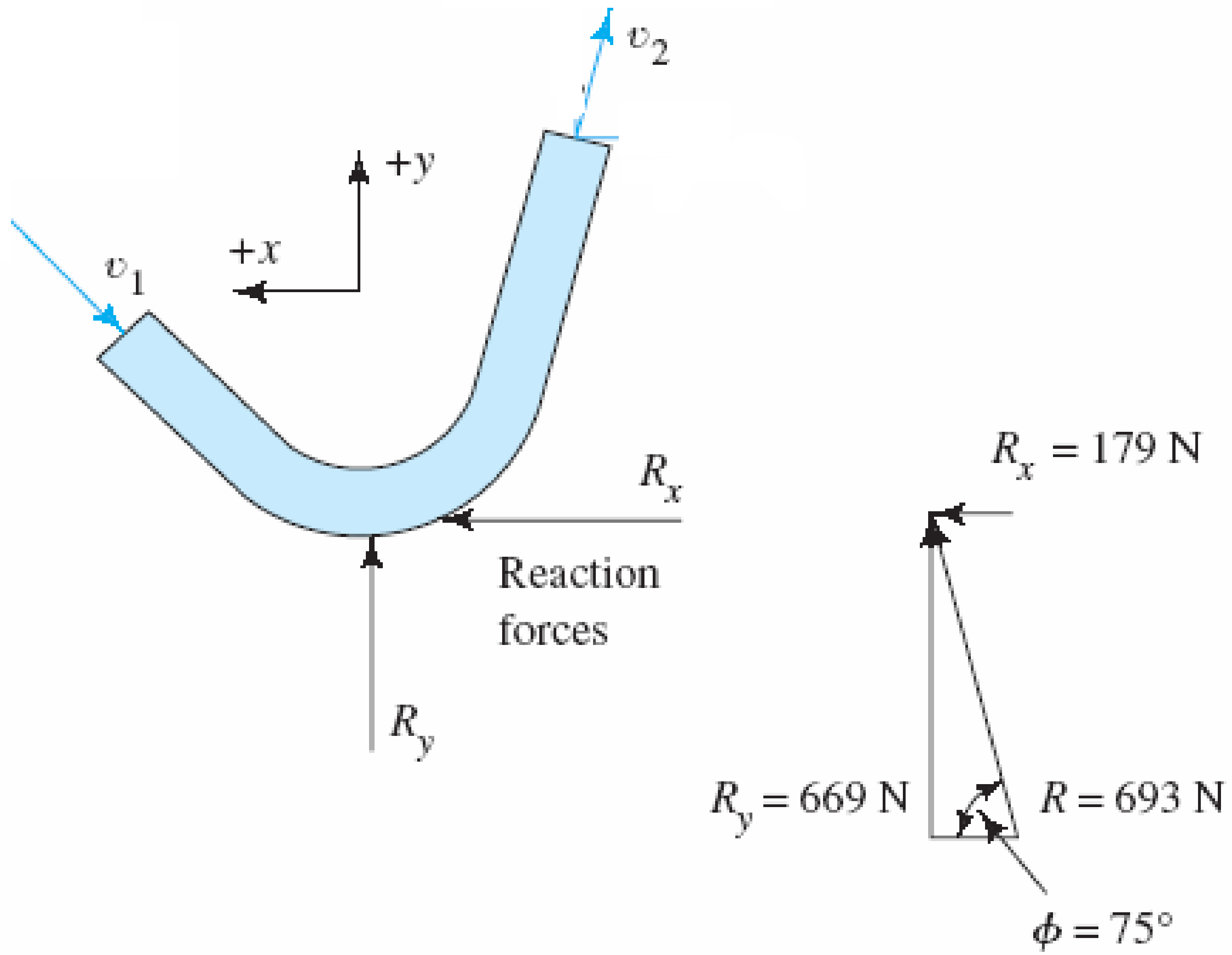


The resultant force

$$F = \sqrt{F_x{}^2 + F_y{}^2}$$

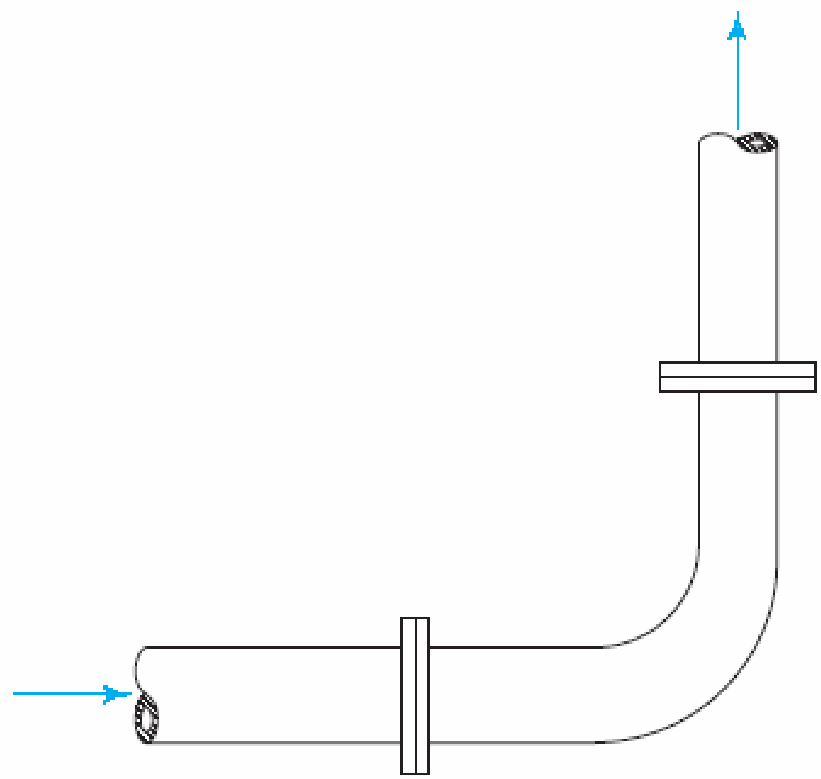
$$F = \sqrt{179.5^2 + 669.5^2}$$

$$F = 693.15N$$



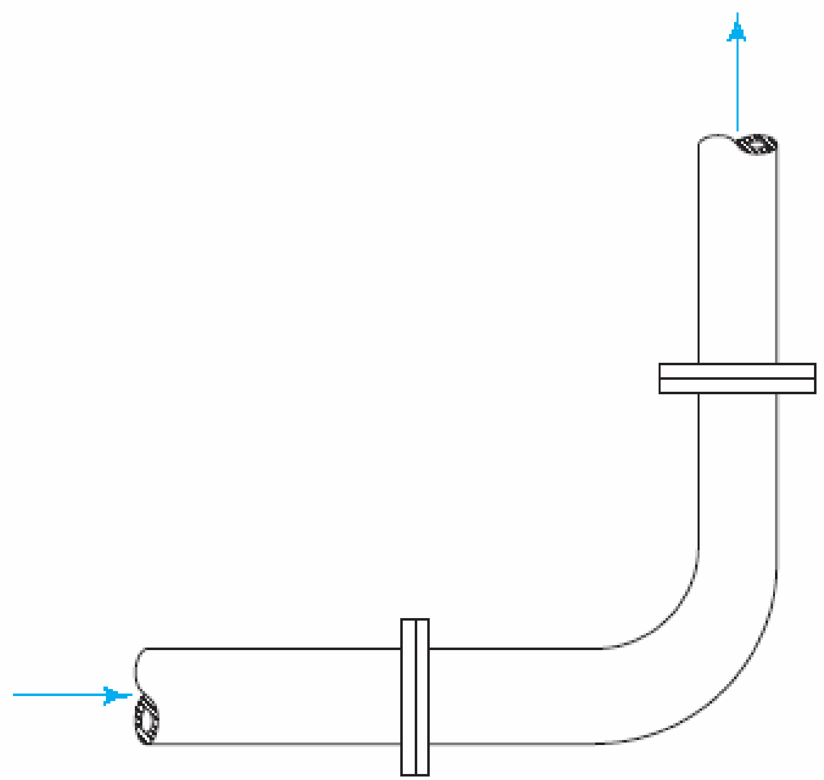
Forces on Bends in Pipeline

- The below figure shows a typical 90° elbow in a pipe carrying a steady volume flow rate *Q*.
- To ensure proper installation, it is important to know how much force is required to hold it in equilibrium.



Example III

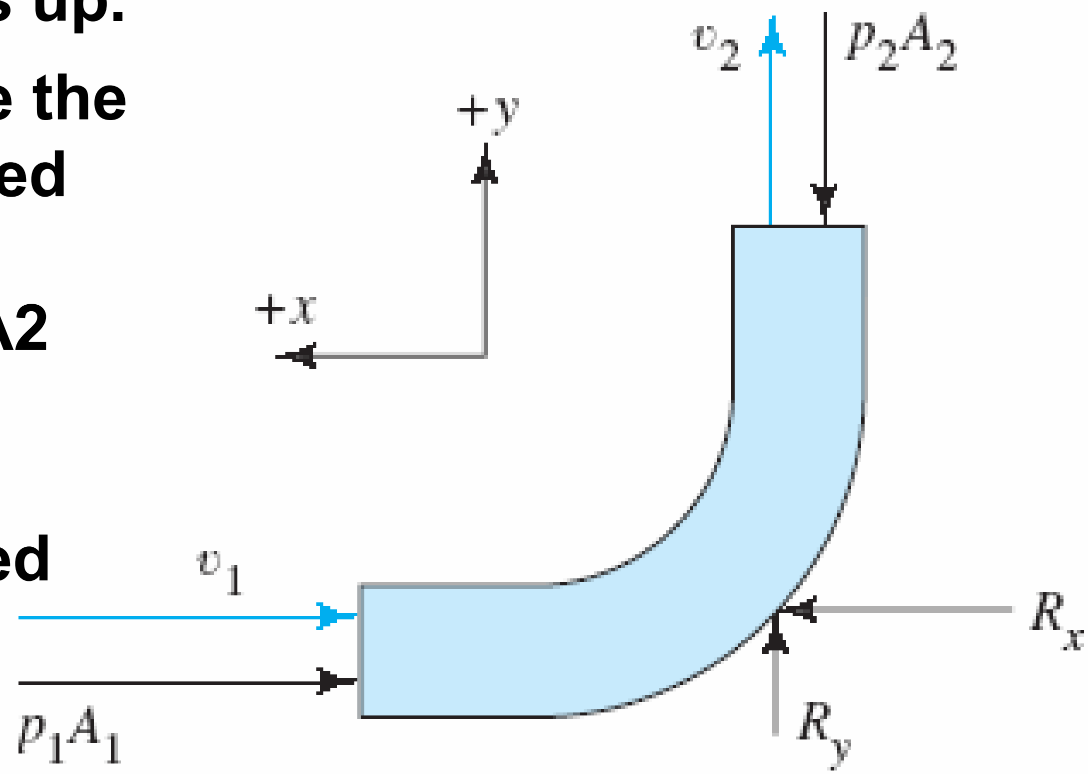
Calculate the force that must be exerted on the pipe shown in the below figure to hold it in equilibrium. The elbow is in a horizontal plane and is connected to two 4-in carrying 3000 L/min of water at 15°C. The inlet pressure is 550 kPa.



The problem may be visualized by considering the fluid within the elbow to be a free body, as shown in Figure. Forces are shown in black vectors, and the direction of the velocity of flow is shown by blue vectors.

A convention must be set for the directions of all vectors. Here we assume that the positive *x* direction is to the left and the positive *y* direction is up.

The forces Rx and Ry are the external reactions required to maintain equilibrium. The forces p1A1 and p2A2 are the forces due to the fluid pressure. The two directions will be analyzed separately



$$F_x = \rho Q(v_{2_x} - v_{1_x})$$

$$F_x = R_x - p_1A_1 \qquad v_{2_x} = 0 \qquad v_{1_x} = -v_1$$

$$R_x - p_1A_1 = \rho Q[0 - (-v_1)]$$

$$R_x = \rho Q v_1 + p_1A_1$$

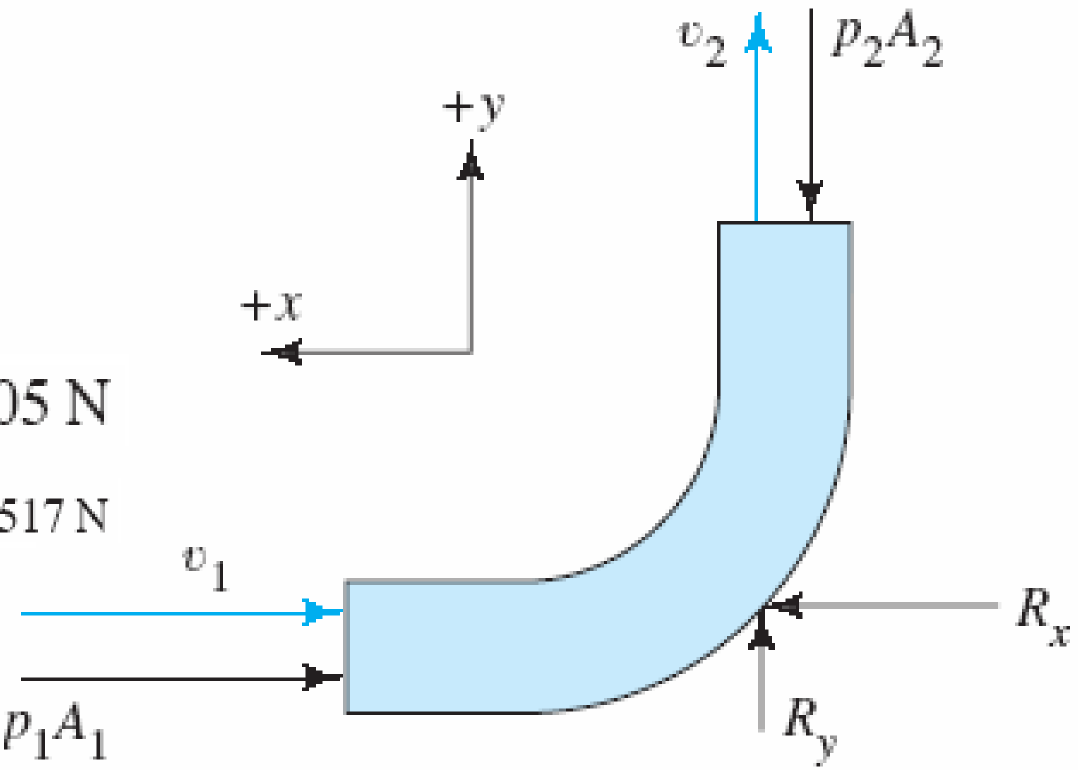
$$Q = 3000\ \text{L/min} = 0.05\ \text{m}^3/\text{s}$$

$$v_1 = \frac{Q}{A_1} = 6.09\ \text{m/s}$$

$$\rho Q v_1 = 1000 \times .05 \times 6.09 = 305\ \text{N}$$

$$p_1A_1 = 550 \times 10^3 \times (8.213 \times 10^{-3}) = 4517\ \text{N}$$

$$R_x = (305 + 4517)\ \text{N} = 4822\ \text{N}$$



In the *y* direction, the equation for the net external force is

$$F_y = \rho Q(v_{2_y} - v_{1_y})$$

$$F_y = R_y - p_2A_2$$

$$v_{2_y} = +v_2$$

$$v_{1_y} = 0$$

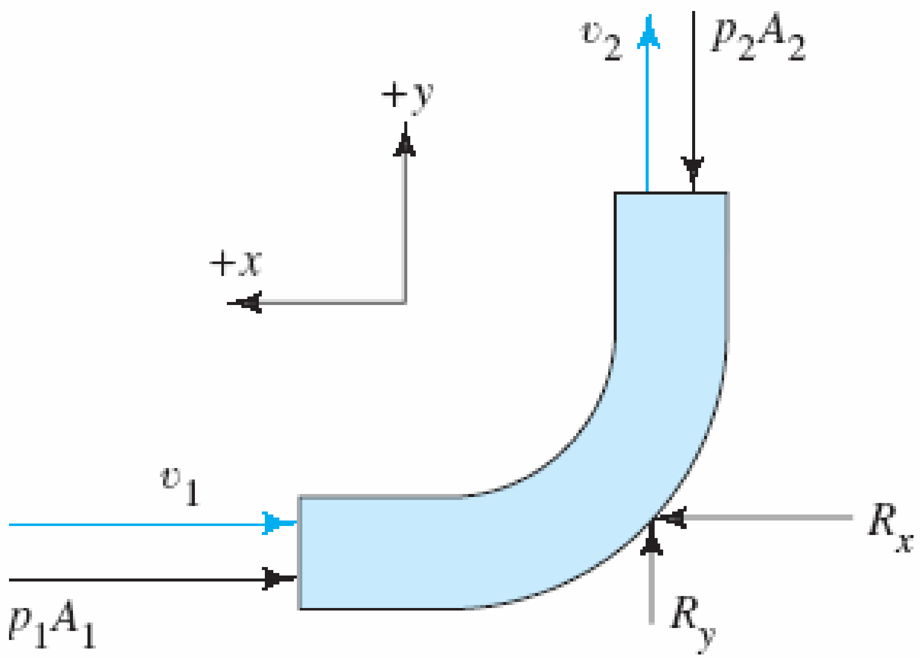
$$R_y - p_2A_2 = \rho Q v_2$$

$$R_y = \rho Q v_2 + p_2A_2$$

$$\rho Q v_2 = 305 \text{ N}$$

$$p_2A_2 = 4517 \text{ N}$$

$$R_y = (305 + 4517) \text{ N} = 4822 \text{ N}$$

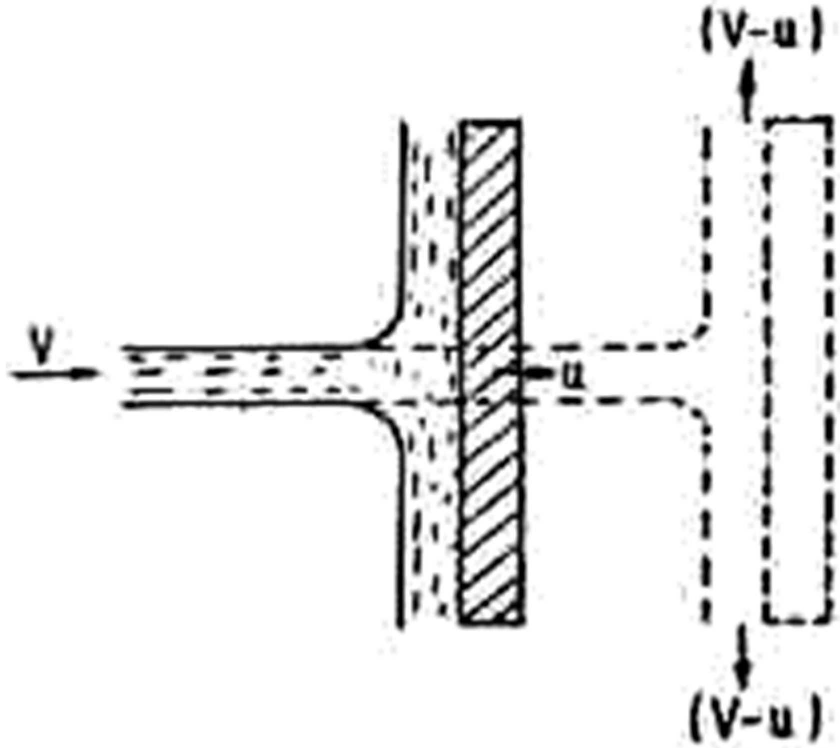


Example IV

A flat plate is struck normally by a jet of water 50mm in diameter with a velocity of 18 m/s. calculate

(a) the force on the plate when it is stationary,

(b) the force on the plate when it moves in the same direction as the jet with a velocity of 6 m/s



Solution

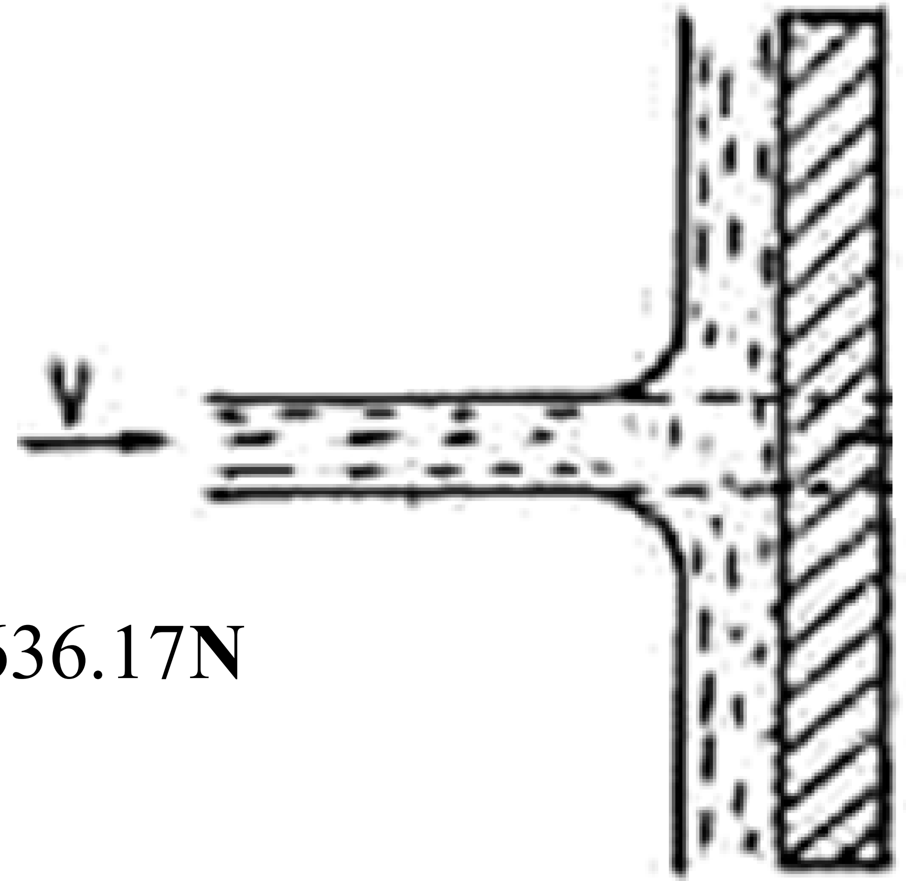
$$\mathbf{Q} = 18 \times \frac{\pi}{4}(0.05)^2 = 0.0353$$

$$\mathbf{F_x} = \mathbf{Q}\rho(0 - \mathbf{v}_1) = -\mathbf{Q}\rho\mathbf{v}_1$$

$$= -0.0353 \times 10^3 \times 18 = -636.17\mathbf{N}$$

$$\mathbf{F_y} = 0$$

$$\mathbf{R} = -\mathbf{F} = 636.17\mathbf{N}$$



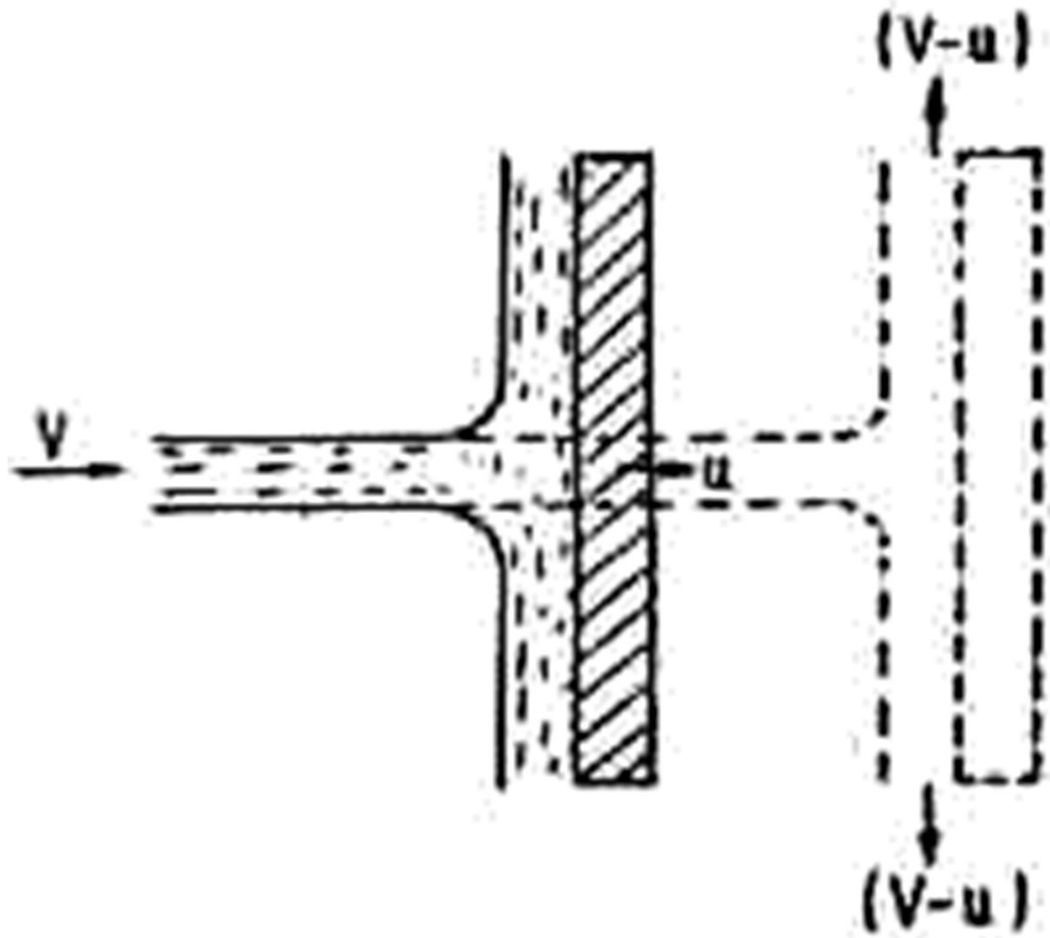
$$\mathbf{Q} = (18-6)\times\frac{\pi}{4}(0.05)^2 = 0.0235\mathbf{m}^3/\mathbf{s}$$

$$\mathbf{F_x} = -\mathbf{Q}\rho(\mathbf{v}_1 - \mathbf{u})$$

$$= -0.0235\times10^3\times(18-6) = -282.4\mathbf{N}$$

$$\mathbf{F_y} = 0$$

$$\mathbf{R} = -\mathbf{F} = 282.4\mathbf{N}$$



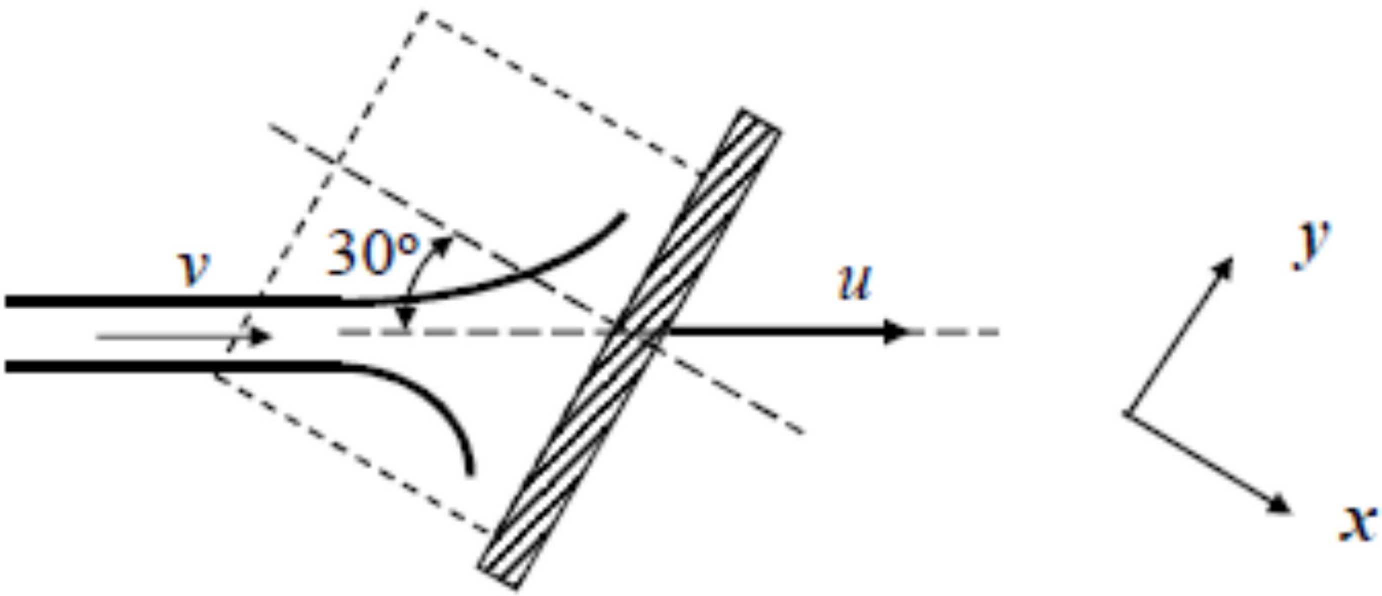
Example V

A jet of water from a fixed nozzle has a diameter d of 25mm and strikes a flat plate at angle u of 30o to the normal to the plate. The velocity of the jet v is 5m/s, and the surface of the plate can be assumed to be frictionless.

Calculate the force exerted normal to the plate

(a) if the plate is stationary.

(b) if the plate is moving with velocity u of 2m/s in the same direction as the jet



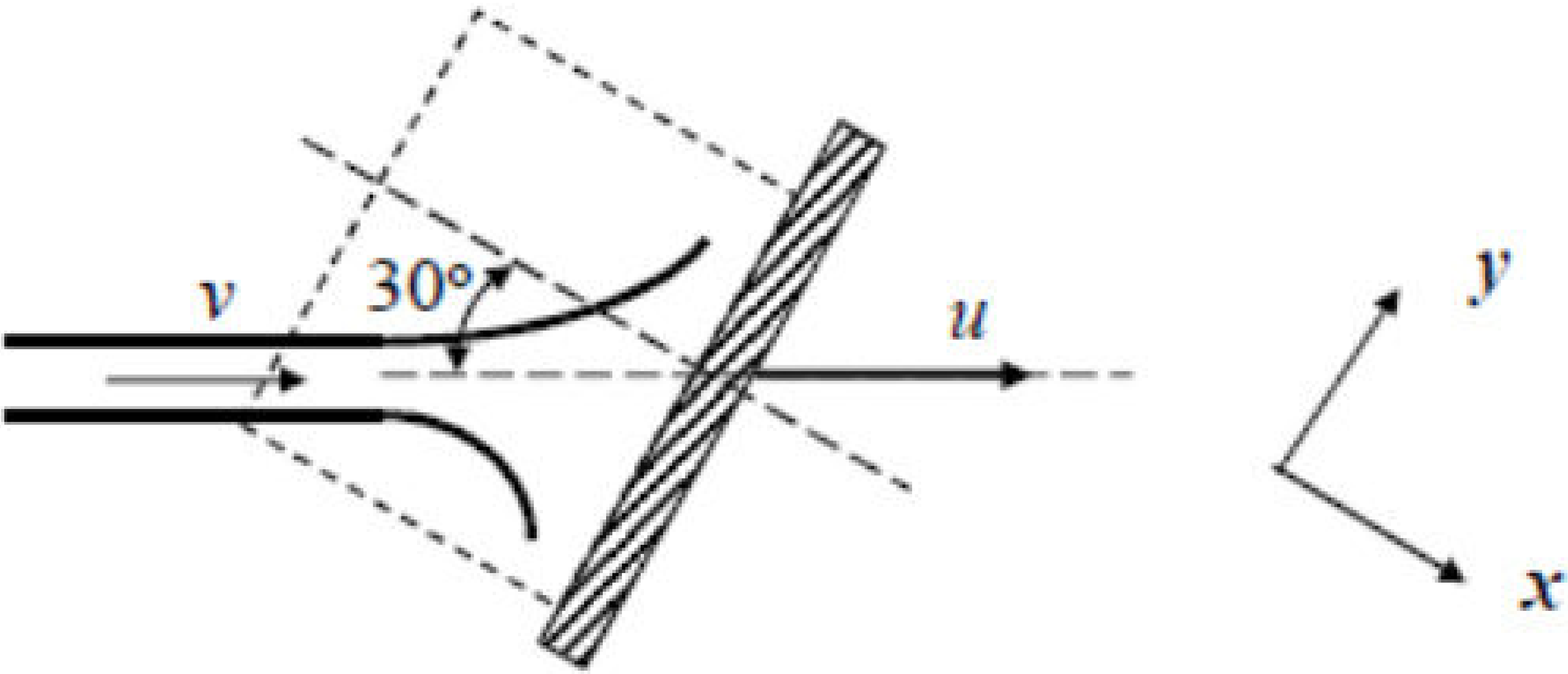
Solution

(a) if the plate is stationary:

- Initial component of velocity relative to plate in x direction = $v\cos\theta$
- Final component of velocity relative to plate in x direction = 0

$$R_x = \dot{m}(v_{in} - v_{out})_x = \rho A v(v\cos\theta) = \rho A v^2 \cos\theta$$

$$R_x = 1000 \times \left(\frac{\pi}{4} 0.025^2\right) \times 5^2 \times \cos 30 = 10.63 N$$



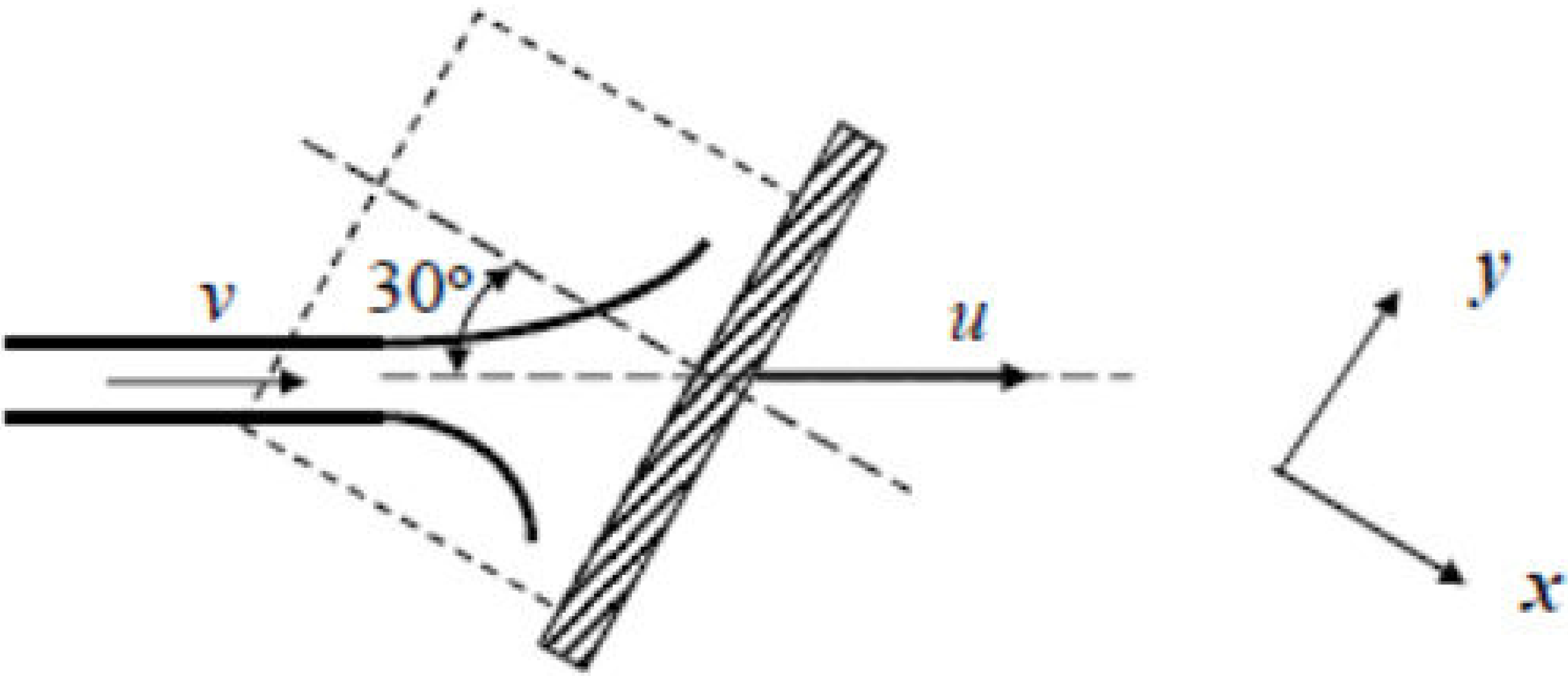
- (b) if the plate move:

Initial component of velocity relative to plate in x direction = $(v-u)\cos\theta$

Final component of velocity relative to plate in x direction = 0

$$R_x = \dot{m}(v_{in} - v_{out})_x = \rho A(v-u)(v-u)\cos\theta = \rho A(v-u)^2\cos\theta$$

$$R_x = 1000 \times \left(\frac{\pi}{4}0.025^2\right) \times (5-2)^2 \times \cos 30 = 3.83N$$



Thank you