MA6459- NUMERICAL METHODS

UNIT-I SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS

(2013-Regulations)

Part – A Questions

1. What are the conditions under which a real root of the equation **f(x)=0** can be found, using simple iteration method?
2. Locate the negative root of $x^3 - 2x + 5 = 0$
3. What do you mean by the order of convergence of an iterative method for finding the root of the equation $f(x) = 0$?
4. State the condition for convergence of Newton-Raphson method.
5. State the order of convergence of (i) the iteration formula and (ii) the Newton-Raphson.
6. Find the iterative formula for square root of N using Newton's method.
7. Find the iterative formula for reciprocal of N using Newton's method.
8. Give any two differences between Gauss Elimination and Gauss Jordan Methods.
9. Distinguish between direct and iterative methods (indirect methods) of solving simultaneous equations.
10. When does Gauss elimination method fail?
11. Sate a sufficient condition for Gauss-Seidel method to converge.
12. How will you find the smallest eigen value of a square matrix, by power method?
13. What are the conditions under which the power method can be applied?
14. When does the power method work satisfactorily?
15. State Fixed point theorem.

PART – B QUESTIONS

1. Find the real root of the equation $x^3+x^2-100=0$ correct to 5 decimal places using Iteration method.
2. Use the method of fixed point iteration to solve the equation **Cos x=3x-1** .
3. Solve $e^x-3x=0$ by the method of fixed point iteration.
4. Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton-Raphson method.

5. Find the real positiveroot of **3x – cosx – 1 =0** by Newton's method correct to 5 decimal places.

6. Find a real root of $x\log_{10} x - 1.2 = 0$ by Newton-Raphson method correct to three decimal places.

7. Solve the system of equations **3x+4y+5z=18, 2x-y+8z=13, 5x-2y+7z=20** by Gauss elimination method.

8. Solve the system of equations **10x-2y+3z=23, 2x+10y-5z = -33, 3x-4y+10z=41** by Gauss elimination method.

9. Using the Gauss-Jordan method solve the following equations **10x+y+z=12, 2x+10y+z=13, x+y+5z=7.**

10. Using Gauss-Jordan method to solve **2x-y+3z=8, -x+2y+z =4, 3x+y-4z =0.**

11. Apply Gauss-Seidal method to solve the system of equations **20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z = 25**.

12. Using Gauss-Seidel method solve the following system of equations **4x+y+2z = 4, 3x+5y+z = 7 and x+y+3z = 3.**

13. Solve thefollowing system of equations by Gauss-Jacobi method.
27x+6y-z = 85, x+y+54z =110, 6x+15y+2z = 72.

14. Find the solution of the system of equations **8x-3y+2z= 20, 6x+3y+12z = 35, 4x+11y-z =33** using Jacobi's iterative method.

15. Using Gauss-Jordan method, find the inverse of A= $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

16. Using Gauss-Jordan method, find the inverse of A = $\begin{bmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & -8 \end{bmatrix}$.

17. Using Gauss-Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$.

18. Find the dominant Eigenvalus and the corresponding Eigen vector of A = $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

19. Find the numerically largest Eigenvalue of A= $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and the corresponding Eigenvector.

20. Using power method, find all the Eigenvalues of A = $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$.

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UNIT-II INTERPOLATION AND APPROXIMATION

Part – A Questions

1. State Newton's formula for interpolation.
2. State Newton's divided difference interpolation formula .
3. State Lagrange's interpolation formula for unequal intervals.
4. What do you mean by inverse interpolation?
5. write down the formula for the cubic spline polynomial **y(x).**
6. State any two properties of divided differences.
7. State any two properties of cubic spline function.
8. Find the divided differences of $f(x)=x^3-x^2+3x+8$ for the arguments **0,1,4,5**.
9. Find the second degree polynomial from the following data:

X	1	2	4
y	4	5	13

10. Given **f(0)= -1 f(1)=1, f(2)=4**, find the roots of Newton's interpolating polynomial equation .

PART – B QUESTIONS

1. Using Lagrange's interpolation formula, find **y(10)** given that **y(5) = 12, y(6) =13, y(9) =14, y(11)=16.**

2. Use Lagrange's method to find $\log_{10} 656$, given that $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$ and $\log_{10} 661 = 2.8202$.

3. Using Lagrange's interpolation, calculate the profit in the year **2000** from the following data:

Year	1997	1999	2001	2002
Profit in lakhs of Rs.	43	65	159	248

4. Find the age corresponding to the annuity value **13.6** given the table:

Age (x)	30	35	40	45	50
Annuity value y)	15.9	14.9	14.1	13.3	12.5

5. Find the value of **x** when **y=20**, using Lagrange's formula from the following table:

x	1	2	3	4
Y=f(x)	1	8	27	64

6. By using Newton's divided difference formula find **f(8),** given

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

7. Find **f(x)** as a polynomial in **x** for the following data by Newton's divided difference formula.

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

8. Using Newton's divided difference formula find **f(x)** and **f(6)** from the following data:

x	1	2	7	8
f(x)	1	5	5	4

9. From the following table:

x	1	2	3
y	-8	-1	18

Compute **y(1.5) and y'(1),** using cubic spline.

10. Obtain the cubic spline approximation for the function **y=f(x)** from the following data, given that $y_0'' = y_3'' = 0$.

x	-1	0	1	2
y	-1	1	3	35

11. Fit a cubic spline curve for the points **(2,11),(3,49)** and **(4,123).** Hence, find **y(2.5)** and $y'(3.5)$. Assume that $y''(2) = 0$ and $y''(4) = 0$.

12. Using Newton's forward interpolation formula, find the cubic polynomial which takes the following values:

x	0	1	2	3
f(x)	1	2	1	10

13 .From the data given below, find the number of students whose weight is between 60 to 70.

Weight in lbs	0-40	40-60	60-80	80-100	100-120
No. of students	250	120	100	70	50

14. Find **y(2.25)** using Newton's backward difference formula from the following data:

x	1.00	1.25	1.50	1.75	2.00
y	0.3679	0.2865	0.2331	0.1738	0.1353

15. From the following table, find the value of $\tan 45^{\circ}15'$ by Newton's forward interpolation formula.

x°	45	46	47	48	49	50
$\tan x^{\circ}$	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

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Unit III – Numerical Differentiation and Integration

Part – A Questions

1. What do you mean by numerical differentiation?
2. Write down the formulas for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at any point in terms of forward differences upto $\Delta^4 y_0$
3. Write down the formulas for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x=x_0 in terms of forward differences upto $\Delta^5 y_0$
4. Write down the formulas for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at any point in terms of backward differences upto $\nabla^4 y_n$
5. Write down the formulas for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x=x_n in terms of backward differences upto $\nabla^5 y_0$
6. Find the first derivative of the function f(x) tabulated below at the point $x = 1.5$

x	1.5	2.0	2.5	3
f(x)	3.375	7.0	13.625	24.0

7. What do you mean by numerical integration?
8. State Simpson's 1/3 and 3/8 formulas for numerical integration.
9. In numerical integration, what should be the number of intervals to apply Simpson's one-third rule and Simpson's three-eighth rule?
10. State the two point Gaussian quadrature formula to evaluate $\int_{-1}^{1} f(x)\,dx$
11. State the three point Gaussian quadrature formula to evaluate $\int_{-1}^{1} f(x)\,dx$
12. Using Simpson's rule, find $\int_{0}^{4} e^x\,dx$ given that $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.6$
13. Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ using Gauss two-point formula.
14. Evaluate $\int_{0}^{\frac{\pi}{2}} \sin t\,dt$ by Gaussian two-point formula.
15. Evaluate $\int_{2}^{3} \frac{dt}{t}$ by Gauss three point formula.
16. State Trapezoidal rule to evaluate a double integral.
17. State Simpson's 1/3 rule to evaluate a double integral.

PART B - QUESTIONS

1. A slider in a machine moves along a fixed straight rod. Its distance X cm along the rod is given below for various values of time 't' seconds. Find the velocity of the slider when t=1.1 second

t	1.0	1.1	1.2	1.3	1.4	1.5	1.6
x	7.989	8.403	8.781	9.129	9.451	9.750	10.031

2. Find the first three derivatives of f(x) at x=1.5 by Newton's forward interolation formula to the data given below.

x	1.5	2	2.5	3	3.5	4
y=f(x)	3.375	7.0	13.625	24.0	38.875	59

3. Given the following data, find $y'(6)$ and the maximum value of y (if it exists)

x	0	2	3	4	7	9
y	4	26	58	112	466	922

4. Evaluate $\int_0^1 \frac{dx}{1+x}$ and correct to three decimal places using Romberg's method and hence find the value of $\log_e 2$.

5. Using Romberg's method to compute $\int_0^1 \frac{dx}{1+x^2}$ correct to four decimal places. Also evaluate the same integral using three- point Gaussian quadrature formula. Comment on the obtained values by comparing with the exact value of the integral which is equal to π/4.

6. Evaluate $\int_0^{\frac{1}{2}} \frac{x}{\sin x}dx$ correct to three decimal places using Romberg's method.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin x\, dx$ using Simpson's 3/8 rule.

8. Evaluate $\int_0^{2\pi} \sin x\, dx$ by dividing the range into 10 equal arts, using (i)Trapezoidal rule (ii) Simpson's one third rule, verify your answer with actual integration.

9. Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^2} dx$ by Gaussian three- point formula.

10. Using Trapezoidal rule, evaluate $\int_1^2\int_1^2 \frac{dxdy}{x^2+y^2}$ numerically with h=0.2 along x direction and k=0.25 along y direction.

11. Evaluate $\int_1^{1.2}\int_2^{2.4} \frac{dxdy}{xy}$ using Simpson's one third rule.

12. Evaluate $\int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}} \frac{\sin(xy)dxdy}{1+xy}$ using Simpson's rule with h=k=0.25.

13. The velocity V of a particle at a distance S from a oint on its path is given by the table below.

S (meter)	0	10	20	30	40	50	60
V(m/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpson's 1/3rd rule and Simpson's 3/8th rule.

14. Use Gauss three-point formula and evaluate $\int_1^5 \frac{dx}{x}$.

15. Evaluate $\int_0^1\int_0^1 \frac{dxdy}{1+x+y}$ by Trapezoidal rule.

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Unit IV – Initial Value Problems for Ordinary Differential Equations

PART A – QUESTIONS

1. What is the difference between an initial value problem and a boundary value problem?
2. What do you mean by truncation error and what is the order of truncation error in Taylor's series method of *n*th order?
3. By Taylor series method, find y(1.1) given $y' = x + y$, $y(0) = 1$
4. Find $y(1.1)$, using Euler's method, from $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1$.
5. State modified Euler algorithm to solve $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$.
6. State Runge-Kutta algorithm of fourth order for the solution of $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$.
7. Compare Taylor series and Runge-Kutta methods.
8. What do we mean by single step methods and multistep methods?
9. Why the predictor-corrector methods are called so?
10. Compare Runge-Kutta method and predictor corrector methods
11. Write down the Milne's predictor-corrector formulae.
12. Write down the Adam-Bashforth's predictor-corrector formulae.

PART B QUESTIONS

1. Use Taylor series method to find y(0.1) and y(0.2) given that $\frac{dy}{dx} = 3e^x + 2y$, y(0)=0 correct to four decimal places.
2. Use Taylor series method to find y(0.1) if $\frac{dy}{dx} = x^2 + y^2$, y(0) = 1.
3. Use Taylor series method find y at x=0 if $\frac{dy}{dx} = x^2 y - 1$, y(0)=1.
4. Solve $\frac{dy}{dx} = \frac{y - x}{y + x}$, y(0)=1 at x= 0.1 by taking h= 0.02 by using Euler's method.

5. Apply modified Euler's method to find y(0.2) and y(0.4) given $\frac{dy}{dx}=x^2+y^2$, y(0)=1 by taking h=0.2.
6. Given $y'+xy'+y=0$, y(0)=1, $y'(0)=0$. Find the value of y(0.1) by using Runge-kutta method of fourth order.
7. Use Runge- kutta method of fourth order to find y(0.2) given $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$, y(0)= 1, taking h=0.2.
8. Use Runge- kutta method to find y(0.2), if $y'=xy'^2-y^2=0$, y(0)=1, $y'(0)=0$, h=0.2.
9. Use Runge- kutta method of fourth order, find the value of y at x= 0.2, 0.4, 0.6 given $\frac{dy}{dx}=x^3+y$ y0)=2. Also find the value of y at x=0.8 using Milne's predictor and corrector method.
10. Use Milne's predictor – corrector formula to find y(0.4). given $\frac{dy}{dx}=\frac{(1+x^2)y^2}{2}$, y(0)=1, y(0.1)=1.06, y(0.2)= 1.12 and y(0.3)= 1.121.
11. Given $\frac{dy}{dx}=xy+y^2$, y(0)=1, y(0.1)=1.1169, y(0.2)=1.2774, y(0.3)=1.5041. Use Adam's method to estimate y(0.4).
12. Use Adam's Bashforth method, find y(4.4) given that $5xy'+y^2=2$, y(4)=1, y(4.1)=1.0049, y(4.2)=1.0097 and y(4.3)=1.0143.
13. Use Adam's predictor corrector method,given that $y'=y-x^2$; y(0)=1, y(0.2)= 1.1218; y(0.4)=1.4682 and y(0.6)=1.7379 ,evaluate y(0.8).
14. Solve for y(0.1) and z(0.1) from the simultaneous differential equations $\frac{dy}{dx}=2y+z$; $\frac{dz}{dx}=y-3z$; y(0)=0, z(0)=0.5 using Runge-kutta method of fourth order.
15. Given that $\frac{dy}{dx}=1+y^2$; y(0.6)=0.6841, y(0.4)=0.4228, y(0.2)=0.2027, y(0)=0, find y(-0.2) using Milne's method.

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Unit V – Boundary Value Problems in Ordinary and Partial Differential Equations

PART A – QUESTIONS

1. Define difference quotient of a function y(x).
2. Obtain the finite difference scheme for the differential equation $2\dfrac{d^2y}{dx^2}+y=5$
3. State the conditions for the equation $Au_{xx}+Bu_{xy}+Cu_{yy}+Du_x+Eu_y+Fu=G$ where *A, B, C, D, E, F* and *G* are functions of *x* & *y*, to be (i) elliptic (ii) parabolic and (iii) hyperbolic .
4. Classify the equation $u_{xx}+4u_{xy}+4u_{yy}-u_x+2u_y=0$
5. For what values of *x* and *y*, the equation $xu_{xx}+yu_{yy}=0$ is elliptic.
6. State Bender-Schmidt's difference equation (or explicit formula) for one dimension heat equation. When does it assume the simplest form?
7. State Crank-Nicholson's difference equation (or implicit formula) for one dimension heat equation. When does it take the simplest form?
8. State explicit finite difference scheme for one dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2}=a^2\dfrac{\partial^2 u}{\partial x^2}$
9. State standard five point finite difference formula for solving $u_{xx}+u_{yy}=0$.
10. Write down the diagonal five point formula to solve the Laplace equation $u_{xx}+u_{yy}=0$.
11. Write down the finite difference formula to solve the Poisson's equation $u_{xx}+u_{yy}=f(x,y)$.
12. Write the Liebmann's iteration process.
13. State the explicit formula for solving heat flow equations.
14. What is the error for solving Laplace and Poisson's equations by finite difference method?
15. For what value of λ , the explicit method of solving the hyperbolic equation $\dfrac{\partial^2 u}{\partial x^2}=\dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$ is stable, where $\lambda=\dfrac{c\Delta t}{\Delta x}$?

PART B – QUESTIONS

1. Solve the difference equation $xy'+y=0,\ y(1)=1,\ y(2)=2$ with h=0.5 and h=0.25.
2. Solve the difference equation $y'-3y'+2y=1+2x,\ y(1)=y(2)=0,$ with h=0.25.

3. Solve the equation $\frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}, 0\le x\le 12, 0\le t\le 12$ with boundary and conditions

$u(x,0)=\frac{1}{4}x(15-x), 0\le x\le 12$, $u(0,t)=0, u(12,t)=9$, $0\le t\le 12$. Using Schmidt relation.

4. Solve $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$, given $u(0, t)=u(5, t)=0$, $u(x,0)=x^2(25-x^2)$, find u in the range taking h=1 and upto 3 seconds using Bender-Schmidt recurrence equation.

5. Find the solution of $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to $u(x,0)=\sin \pi x; 0\le x\le 1$, $u(0, t)=u(1, t)=0$ using Schmidt method.

6. Solve by Crank-Nicolson's method $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ for 0<x<1, t>0 given that $u(0, t)=0$, $u(1, t)=0$ and $u(x,0)=100x(1-x)$. Compute u for one time step with h=1/4 and K=1/64.

7. Solve $u_{tt}=4u_{xx}$ with boundary conditions $u(0,t)=0=u(4,t), u_t(x,0)=0 \;\&\; u(x,0)=x(4-x)$.

8. Solve $25u_{xx}-u_{tt}=0$ for u at the pivotal points given $u(0,t)=0, u(5,t)=0, u_t(x,0)=0$ and $u(x,0)=\begin{cases} 2x, 0\le x\le 2.5 \\ 10-2x, 2.4\le x\le 5 \end{cases}$ for one half period of oscillation taking h=1.

9. Solve: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, 0<x<1, t>0 given $u(x,0)=0, u(0, t)=0$, $u_t(x,0)=0$ & $u(1, t)=100\sin \pi t$ compute u for 4 times steps with h=0.25.

10. By iteration method, solve the elliptic equation $u_{xx}+u_{yy}=0$ over a square region of side 4, satisfying the boundary conditions.

    (i) $u(0, y)=0, 0\le y\le 4$ (ii) $u(4, y)=12+y, 0\le y\le 4$

    (iii) $u(x,0)=3x, 0\le x\le 4$ (iv) $u(x,4)=x^2, 0\le x\le 4$.

    By dividing the square into 16 square meshes of side 1 and always correcting the computed values to two places of decimals, obtain the values of u at 9 interior pivotal points.

11. Solve $u_{xx}+u_{yy}=-81xy$, 0<x<1, 0<y<1, choose h=1/3, u(0,y)=u(x,0)=0, u(1,y)=u(x,1)=100.

12. Solve $u_{xx} + u_{yy} = 0$, $0 \le x \le 4, 0 \le y \le 4$;

$u(x,0) = x^2 + 2x; u(0, y) = -y^2 - 2y, u(4, y) = 24 - y^2 - 2y; u(x,4) = x^2 + 2x - 24$ with h=1.

13. Solve the Poisson equation $\nabla^2 u = -10(x^2 + y^2 + 10), 0 \le x \le 3, 0 \le y \le 3, u = 0$ on the boundary.

14. Solve $\nabla^2 u = 8x^2 y^2$, on the 4 boundaries dividing the square into16 sub squares of length 1 unit.

15. Solve $\nabla^2 u = -1, |x| < 1, |y| < 1$, $u(\pm 1, y) = u(x, \pm 1) = 0$, choose h=1/2.

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